Using pedagogical maps to show the opportunities afforded by CAS for improving the teaching of mathematics

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Experience and research in teaching secondary school mathematics with Computer Algebra Systems (CAS) has been building over the past two decades. For Australian experience see for example Ball & Stacey (2005), Flynn (2003), Garner (2004), Geiger (2003), Goos et al. (2003), Kendal & Stacey (2001), Leigh-Lancaster (2002), Pierce (2001), Pierce & Stacey (2002) Stacey (2005), Tynan, (2003). From this experience we have come to see that CAS affords a range of key opportunities to change and improve the teaching of mathematics. Teachers' perceptions of these affordances vary greatly. Just as there is great variety in the "traditional" non CAS classroom environment, when CAS is available different teachers make different choices about the changes they wish to make to their teaching style and approach to mathematics.

Figure 1 shows what we call a pedagogical map, developed by the present authors and first presented by Stacey (2007). It provides a framework for considering the range of opportunities that CAS may offer for changing the practice of teaching and learning mathematics. At the base of the figure, the box shows that the affordances of CAS for teaching depend on its intrinsic characteristics. Its primary purpose is to support mathematicians to work on non trivial problems involving tedious or impractical levels of calculations and manipulations and as such it executes standard algorithms quickly and correctly.

Also arising from these intrinsic capabilities are the possibilities for making change to the content of the curriculum and the possibility of changing the assessment significantly. These are not discussed in this paper. Instead the focus of Figure 1, and of this paper, is the way in which CAS offers pedagogical opportunities — for teaching mathematics better and for learning mathematics better.

The three rows of the pedagogical map (Figure 1) show three different types of opportunities. On the bottom row are the opportunities most closely related to the speed, accuracy and access to different mathematical representations (numeric, symbolic, graphic). The middle row draws attention to the fact that CAS may be a catalyst for changes to the dynamics of a classroom.

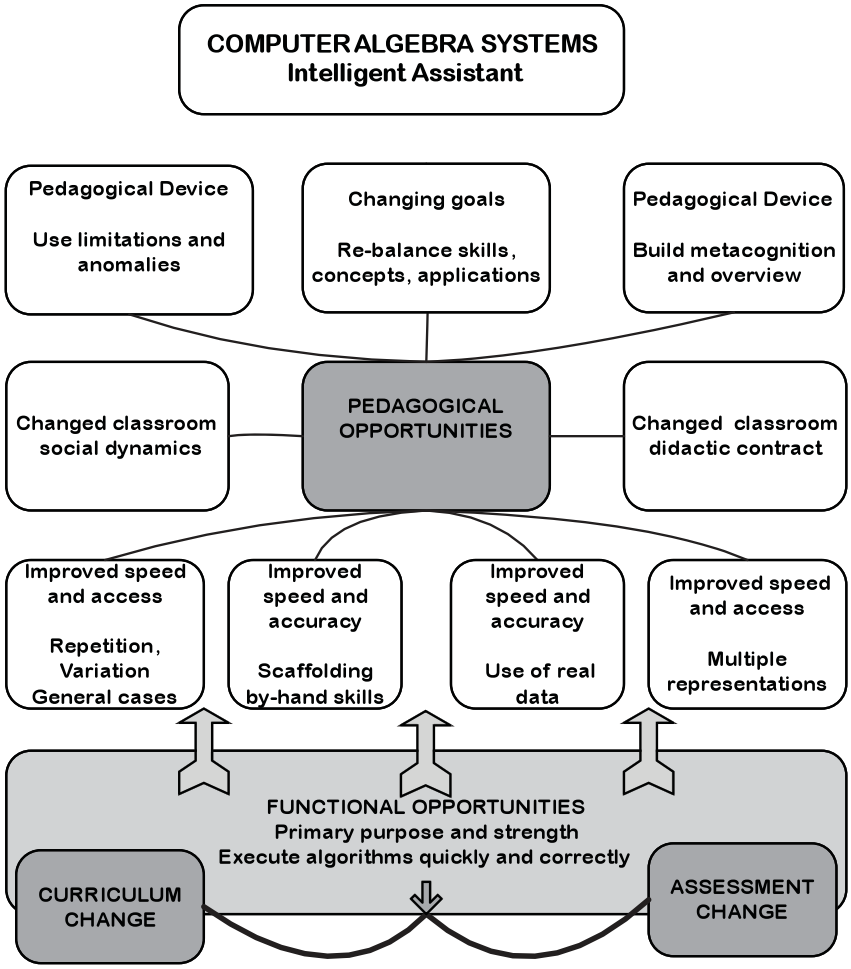

Figure 1. Pedagogical map of opportunities afforded by CAS for improving the teaching of mathematics.

Students need no longer rely on the teacher as the source of mathematical authority and with help they can learn to use CAS as a tool to support their own individual and shared learning. On the top row, we highlight opportunities for deeper learning which a teacher may choose to tap into. The limitation of the technology, and consequent anomalies encountered, may be swept aside or used as a stimulus for rich mathematical discussion. The functionality of CAS which allows the use of multiple representations, and trivialises the manipulation of symbolic expressions, may facilitate the presentation of an overview of a mathematical topic before or after engagement with the detail. In the centre of the row, the diagram highlights the way in which goals may be changed through a rebalancing of concepts, skills and applications. Too often secondary mathematics is dominated by skills used with little conceptual understanding or sense of purpose and value. Stacey (2007) gave illustrations of each of the 8 boxes.

Teaching with CAS: Three experiences of practice

In the sections below we describe the practice of three different teachers in terms of this framework. These scenarios are based on classroom teachers

whose work we have observed in different studies over the past decade. Highlighting the different pedagogical uses made by the teachers on the pedagogical map gives us a visual representation of differences between teaching practices (see Figures 2, 3 and 4).

Classic

Many early adopters of CAS for teaching concentrated on the "functional" opportunities CAS provides. CAS were designed to assist mathematicians: to save time and reduce simple manipulation errors. A fundamental strength of CAS is that they will execute algorithms quickly and correctly.

Teacher 1 was responsible for a first year undergraduate course covering functions, early calculus and linear algebra. Initially this teacher saw value in the students' learning to use CAS to supplement their by-hand work, because technology would allow the students access to more real world "messy" problems. Improved speed and accuracy with CAS would provide scaffolding for students' by-hand skills and support the use of real world data. Except for the time spent using technology instead of pen and paper, this use of CAS did not disturb the usual pattern of teaching but it did extend the range of problems. Later, in the light of early international studies (see for example Heid, 1988), Teacher 1 decided to explore the use of CAS to support students' learning of mathematics. Instead of focusing on careful exposition by the teacher, students were set tasks

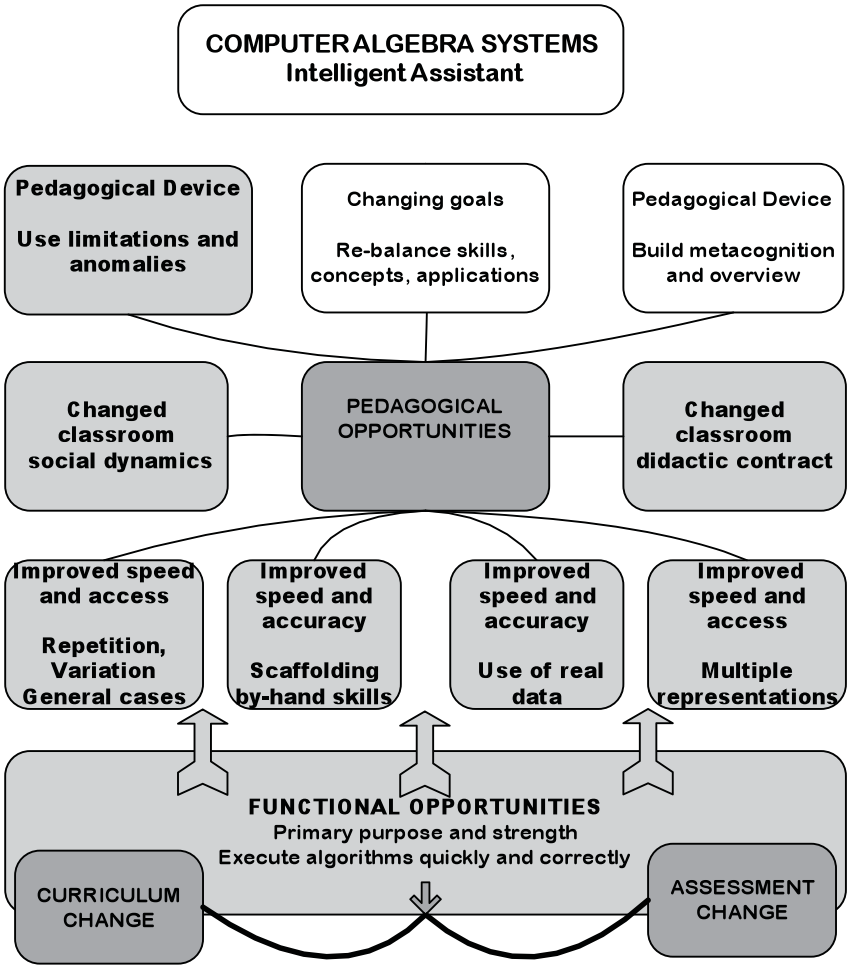

Figure 2. Classic teacher scenario.

which required them to explore new mathematical concepts through the use of multiple representations (symbolic, graphic and numeric) and systematically varying parameters, thus utilising all the affordances on the third row. This required a change in the social dynamics of the classroom: students often worked in pairs or small groups; the teacher was not the only expert in the classroom; and the CAS came to be treated as a second authority (a second row affordance). The didactic contract changed as students were expected to take more responsibility for their own learning, to experiment, try different methods and to share their results. Limitations and anomalies encountered by students (for example misrepresentation of the graph of a function due to pixilation issues) provided opportunity for rich mathematical discussion. During the period of our study the curriculum remained the same but assessment changed to allow access to CAS for all tasks except one in class basic skills test. CAS use was permitted for examinations and assignments. By the end of our study, this teacher was making pedagogical use of CAS in seven different ways.

Progressive

Teacher 2 was teaching senior secondary mathematics and we have classified their approach to the use of CAS in the classroom as "progressive" because they were willing to engage with new technology and prepare their students for CAS permitted examinations. In their situation the curriculum and assess-

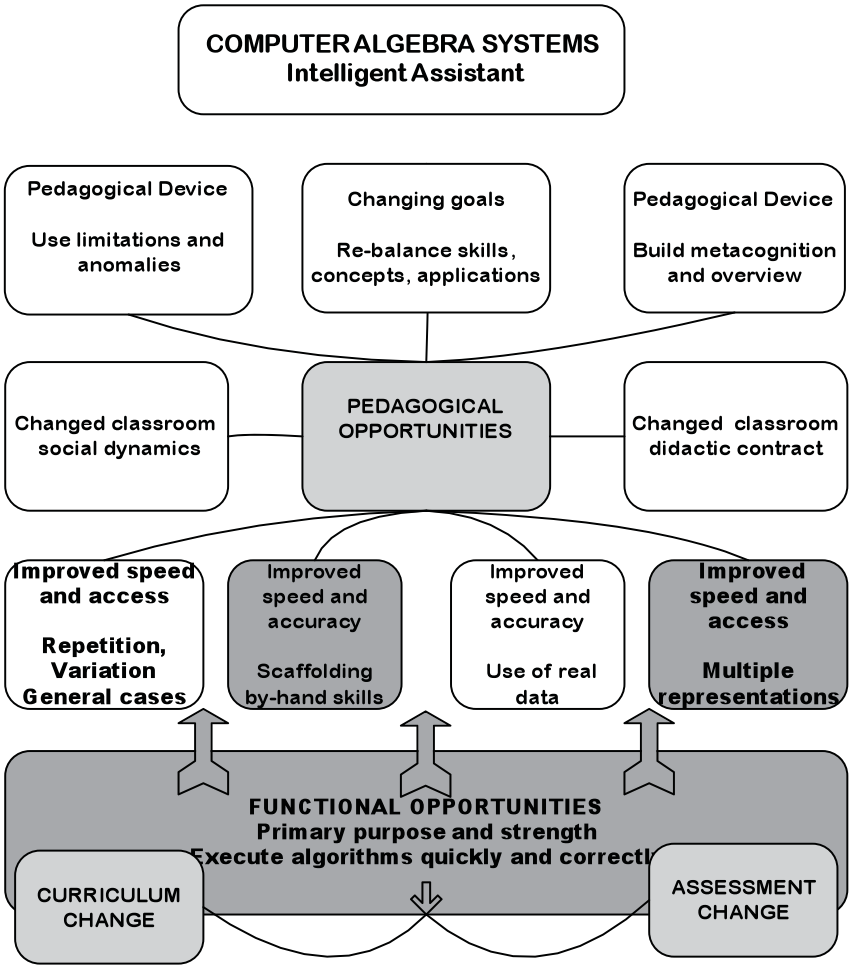

Figure 3. Progressive teacher scenario.

ment were set externally. There had been a limited change in curriculum to include some new topics and a broader range of functions to be studied. However, there was a major change in assessment as the use of CAS was permitted for all assessment tasks, including examinations.

Teacher 2 valued the use of CAS for speed, checking and dealing with "messy" problems. She focused on the functional opportunities provided by CAS and saw the key role of CAS as scaffolding students' by-hand skills. This teacher remained the source of intellectual authority in the classroom. Their pattern of teaching remained fundamentally unchanged as they taught by-hand first then later allowed students to replicate these processes using CAS. This teacher exhibited only two of the pedagogical uses of CAS.

Radical

Teacher 3 was teaching senior secondary mathematics in the same system as Teacher 2: there had been a limited change in curriculum to include some new topics and a broader range of functions to be studied. However, there was a major change in assessment as the use of CAS was permitted for all assessment tasks including examinations. We have described Teacher 3 as radical because she saw the introduction of CAS to the classroom as an opportunity to reflect on, and radically change, her approach to teaching mathematics. Teacher 3 primarily valued the pedagogical opportunities afforded by CAS.

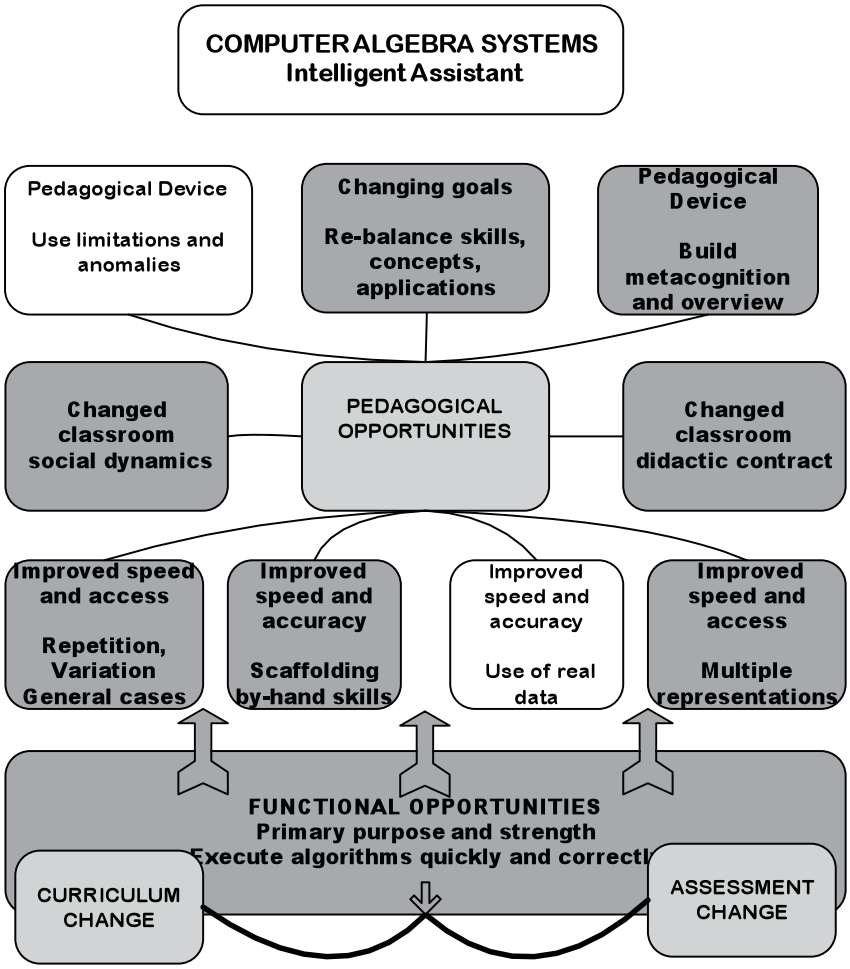

Figure 4. Radical teacher scenario.

Rather than starting at the beginning of a topic, as typically set out in the text book, Teacher 3 approached different topics in different ways. Sometimes, with the help of CAS, she would take her students on a magic carpet ride over the full topic, perhaps starting with an interesting application or what would "normally" have been seen as an outcome of a process. The class would then revisit the detail with a mix of by-hand and by-CAS exercises, teacher exposition and sharing of the results of students' exploration. This is indicated by the shading of one of the boxes on the top row of the pedagogical map.

In Teacher 3's classroom the didactic contract and social dynamic changed. In this class students were expected to explore mathematical possibilities and concepts by considering multiple representations, varying parameters and establishing patterns for dealing with general cases. Students were encouraged to share their findings and solution strategies. This included having students take over the CAS, which was projected for the rest of the class to see. The student would then explain their technical and mathematical strategies. As a result, an explosion of methods occurred with some students showing a preference for using CAS and other for working by hand and most for a mixture. Teacher 3 led discussion about the efficiency of different solution methods and delighted in the individual choices students made regarding by-hand or by-CAS and use of different representations; symbolic, graphic and numeric.

Teacher 3 certainly taught the students to appreciate the functional opportunities afforded by CAS, encouraging them to be discerning about when CAS would offer greater speed and accuracy. Such decision making would be important in examination contexts.

Conclusion

In the scenarios above we see three teachers using CAS in quite different ways. The pedagogical maps derived from Figure 1 illustrate this. These differences may result from their beliefs about mathematics and the culture they prefer to foster in their classroom. On the other hand it may be a result of their experience or inexperience in teaching with technology, in general, and with CAS, in particular. Just as students need to learn to work effectively with CAS so do teachers need to learn to teach effectively with CAS and avail themselves, and their students, of the opportunities for improved ways of both doing and learning mathematics which CAS may afford.

The pedagogical maps provide a graphic depiction of the way in which teachers exploit the presence of CAS in their classrooms. They can show differences, and can also show growth, as well as personal preferences and reactions to particular teaching assignments. They might also be used for teachers to reflect on their own practice of teaching with technology and to set goals for expanding their pedagogical repertoires.

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